$+\cot^{-1}(a/d)]-\frac{1}{4}b^2[\cot^{-1}(b/c)+\cot^{-1}(b/d)]+\frac{1}{4}c^2[\tan^{-1}(a/c)-\tan^{-1}(b/c)]$

$$+\tfrac{1}{4}d^2[\tan^{-1}(a/d)-\tan^{-1}(b/d)]-\tfrac{1}{4}a(c+d)+\tfrac{1}{4}b(c+d)$$

$$=\tfrac{1}{4}a^2\cot^{-1}\left(\frac{a^2-cd}{ac+ad}\right)-\tfrac{1}{4}b^2\cot^{-1}\left(\frac{b^2-cd}{bc+bd}\right)+\tfrac{1}{4}c^2\tan^{-1}\left(\frac{ac-bc}{ab+c^2}\right)$$

$$+\tfrac{1}{4}d^2\tan^{-1}\left(\frac{ad-bd}{ab+d^2}\right)-\tfrac{1}{4}(a-b)(c+d).$$

(*b*) Let $r=4a\cos\theta/\sin^2\theta$....(1), $r=4b\cos\theta/\sin^2\theta$....(2), $r=4c\sin\theta/\cos^2\theta$....(3), $r=4d\sin\theta/\cos^2\theta$....(4), be the equations to the parabolas; $a>b$, $c>d$.

Let $\tan^{-1}\sqrt[3]{(a/c)}=\theta'$.

$$A=8a^2\int_{\theta'}^{\frac{1}{2}\pi}\frac{\cos^2\theta}{\sin^4\theta}d\theta+8c^2\int_0^{\theta'}\frac{\sin^2\theta}{\cos^4\theta}d\theta=\tfrac{16}{3}ac.$$

Similarly, $B=\frac{16}{3}ad$, $C=\frac{26}{3}bc$, $D=\frac{16}{3}bd$.

Area $ABCD=A+B-C-D=\frac{16}{3}(a-b)(c+d)$.

113. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, Ohio.

Deduce the Sylvestrian Reciprocant from $x^4+y^4=4x^2y^2$.

Solution by the PROPOSER.

The given equation may be written $(x^2-y^2)^2=2x^2y^2$.

$\therefore x^2-y^2=xy\sqrt{2}$....(*a*).

Put $(y/x)=w$; then from (*a*), by dividing by xy, etc., we get $w-w^{-1}=-\sqrt{2}$.

$\therefore\ w=\frac{1}{2}(-\sqrt{2}\pm\sqrt{3}),=m$....(*b*).

$\therefore y=mx$, and $dy/dx=m$....(*c*).

Eliminating m from (*c*) by a second differentiation, we have $d^2y/dx^2=0$ ....(*d*).

Adopting Professor Sylvester's notation for reciprocants, viz: $dy/dx=t$, $d^2y/dx^2=a\ 2!$, $d^2y/dx^3=b\ 3!$, etc., we obtain from (*d*) the *first* pure Sylvestrian Reciprocant. All reciprocants *not* containing $t=dy/dx$ are *pure*; all others are *mixed*. The first two pure reciprocants are a and $4ac-5b^2$, and the first two mixed ones are $(1+t^2)b-2a^2t$ and $bt-a^2$. See our paper on Reciprocants published in the MONTHLY of November, 1894.

Also solved by *G. B. M. ZERR*.

114. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

When the sun's declination was 15° N. his altitude was found to be 20°, and after an hour's interval his altitude was found to be 31°. Required, the latitude of the place of observation.